Board Paper of Class 12-Science Term-I 2021 Math Delhi(Set 4) - Solutions

Total Time: 90

Total Marks: 40.0

Section A

Solution 1

Let $y = \log\left[\log\left(\log x^5\right)\right]$

$\frac{dy}{dx} = \frac{d}{dx}\left(\log\left[\log\left(\log x^5\right)\right]\right)$

$= \frac{1}{\log(\log x^5)}\frac{d}{dx}\left(\log\left(\log x^5\right)\right) \qquad \left(\because \frac{d}{dx}(\log x) = \frac{1}{x}\right)$

$= \frac{1}{\log(\log x^5)(\log x^5)}\frac{d}{dx}\left(\log\left(x^5\right)\right)$

$= \frac{1}{\log(\log x^5)(\log x^5)x^5}\frac{d}{dx}\left(x^5\right)$

$= \frac{5x^4}{\log(\log x^5)(\log x^5)x^5} \qquad \left(\because \frac{d}{dx}(x^n) = nx^{n-1}\right)$

$= \frac{5}{x(\log x^5)\log(\log x^5)}$

Hence, the correct answer is option (a).

Solution 2

Number of elements in a 2×3 matrices = 6
Now, 1 element can be filled in two ways either 1 or 2.
Therefore, the number of ways in which one element can be filled = 2^1
Number of ways in which 6 elements can be filled = 2^6 = 64
Thus, the total number of matrices with each element 1 or 2 is 64.

Hence, the correct answer is option (c).

Solution 3

Given that, $f(x) = x^3 + 1$

$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left(x^3+1\right)$

$= 3x^2$

Now put $f'(x) = 0$

$\Rightarrow 3x^2 = 0$

$\Rightarrow x = 0$

Now, for maxima or minima,

$\frac{d^2 f(x)}{dx^2} = 6x$

Since, $\frac{d^2 f(x)}{dx^2}$ at $x = 0$ is 0.

Thus, the double derivative test fails.

Now for $x < 0$, $f'(x) > 0$ and for $x > 0$ also $f'(x) > 0$.

Therefore, $f(x) = x^3 + 1$ will neither have maxima nor will have minima.

Hence, the correct answer is option (d).

Solution 4

Given : $\sin y = x\cos(a+y)$

Differentiating both sides with respect to *x*.

$\sin y = x\cos(a+y)$

$\Rightarrow x = \frac{\sin y}{\cos(a+y)}$

Differentiating with respect to y, we get

$$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{\cos(a+y)\frac{\mathrm{d}\sin y}{dy} - \mathrm{siny}\frac{\mathrm{d}\cos(a+y)}{dy}}{\cos^2(a+y)} \qquad (\because \text{ By quotient rule})$$

$$= \frac{\cos(a+y)\cos y - \sin y(-\sin(a+y))}{\cos^2(a+y)}$$

$$= \frac{\cos(a+y)\cos y + \mathrm{siny}\sin(a+y)}{\cos^2(a+y)}$$

$$= \frac{\cos(a+y-y)}{\cos^2(a+y)} \qquad (\because \cos(x)\cos(y) + \sin(x)\sin(y)\ ) = \cos(x$$

$$= \frac{\cos(a)}{\cos^2(a+y)}$$

Hence, the correct answer is option (a).

Solution 5

Differentiating with respect to x

$\frac{2x}{9}+\frac{2y}{25}\frac{\mathrm{d}y}{\mathrm{d}x}=0$
$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{25x}{9y}$

Now, the tangent is parallel to x-axis, if the slope of the line is equal to zero.
$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=0$
$\Rightarrow \frac{-25x}{9y}=0$
$\Rightarrow x=0$
$\Rightarrow y^2=25$
$\Rightarrow y=\pm 5$

Thus, the points at which tangent is parallel to x-axis is $(0,\pm 5)$.

Hence, the correct answer is option (b).

Solution 6

Three points $(x_1,\ y_1),\ (x_2,\ y_2)$ and $(x_3,\ y_3)$ are collinear if

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} 2x & x+3 & 1 \\ 0 & x & 1 \\ x+3 & x+6 & 1 \end{vmatrix}=0$$

$\Rightarrow 2x\,(x-x-6)-(x+3)\,(0-x-3)+1\left(0-x^2-3x\right)=0$
$\Rightarrow -12x+x^2+6x+9-x^2-3x=0$
$\Rightarrow -9x+9=0$
$\Rightarrow x=1$

Hence, the correct answer is option (d).

Solution 7

Let $\cos^{-1}\left(\frac{1}{2}\right)=x$
$\Rightarrow \frac{1}{2}=\cos x$
$\Rightarrow \cos\left(\frac{\pi}{3}\right)=\cos x$
$\Rightarrow x=\frac{\pi}{3}$

Also, let $\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right) = y$

$\Rightarrow -\frac{1}{\sqrt{2}} = \sin y$

$\Rightarrow \sin\left(-\frac{\pi}{4}\right) = \sin y$

$\Rightarrow y = -\frac{\pi}{4}$

$\therefore$ Principal value of $\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{-1}{\sqrt{2}}\right) = x + y$

$= \frac{\pi}{3} + \left(-\frac{\pi}{4}\right)$

$= \frac{\pi}{12}$

Hence, the correct answer is option (a).

Solution 8

Given: $(x^2 + y^2)^2 = xy$

Differentiating both sides with respect to *x*, we get

$2\left(x^2 + y^2\right) \times \left(2x + 2y\frac{dy}{dx}\right) = x\frac{dy}{dx} + y$

$\Rightarrow 4\left(x^2 + y^2\right)\left(x + y\frac{dy}{dx}\right) = x\frac{dy}{dx} + y$

$\Rightarrow 4\left[\left(x^2 + y^2\right)(x) + \left(x^2 + y^2\right)\left(y\frac{dy}{dx}\right)\right] = x\frac{dy}{dx} + y$

$\Rightarrow 4\left[(x)\left(x^2 + y^2\right) + \left(x^2 + y^2\right)\left(y\frac{dy}{dx}\right)\right] = x\frac{dy}{dx} + y$

$\Rightarrow \left[4\left(x^2 + y^2\right)\left(y\frac{dy}{dx}\right) - x\frac{dy}{dx}\right] = y - 4x\left(x^2 + y^2\right)$

$\Rightarrow \left[4(y)\left(x^2 + y^2\right) - x\right]\frac{dy}{dx} = y - 4x\left(x^2 + y^2\right)$

$\Rightarrow \frac{dy}{dx} = \frac{y - 4x\left(x^2 + y^2\right)}{4y(x^2 + y^2) - x}$

Hence, the correct answer is option (c).

Solution 9

If matrix *A* is symmetric then,

$A^T = A$ .....(1)

If matrix *A* is skew-symmetric then,

$A^T = -A$ .....(2)

And, also the diagonal elements are zero.

Now, it is given that matrix A is both symmetric and skew-symmetric.
$\therefore A^T = A = -A$

This is possible only when *A* is a zero square matrix.

Hence, the correct answer is option (b).

Solution 10

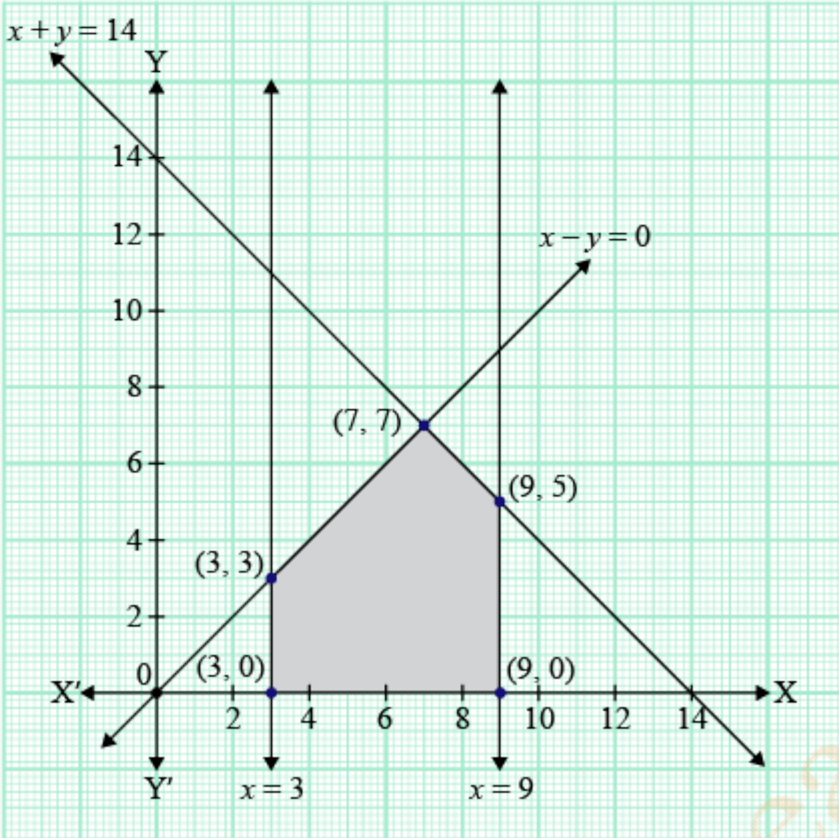
Given: $X = \{1, 2, 3\}$ and $R = \{(1, 3), (2, 2), (3, 2)\}$
To make R reflexive (1, 1) and (3, 3) need to be added and to make *R* symmetric (3, 1) and (2, 3) need to be added.
Thus, a minimum of 4 order pairs, i.e., {(1, 1), (3, 3), (3, 1), (2, 3)} need to be added to make *R* both reflexive and symmetric.

Hence, the correct answer is option (c).

Solution 11

The given linear programming problem is,
Minimize $Z = 2x + y$ subject to
$x \geq 3$,
$x \leq 9$,

$y \geq 0$,
$x - y \geq 0$,
$x + y \leq 14$
The feasible region determined by the given constraints can be graphically represented as,



Clearly, there are 5 corner points including (7, 7) and (3, 3).
Hence, the correct answer is option (b).

Solution 12

For f to be continuous at $x = 0$,

$$\lim_{x\to 0} f(x) = f(0)$$

$$\Rightarrow \lim_{x\to 0} \frac{e^{3x} - e^{-5x}}{x} = k$$

$$\Rightarrow \lim_{x\to 0} \frac{8e^{-5x}\left(e^{8x} - 1\right)}{8x} = k$$

$$\Rightarrow k = 8 \qquad \left(\lim_{x\to 0} \frac{e^{x} - 1}{x} = 1\right)$$

Hence, the correct answer is option (d).

Solution 13

$$C_{31}=\begin{vmatrix} -1 & 2 \\ 2 & -3 \end{vmatrix}$$
$$=(-1)\times(-3)-2\times 2$$
$$=3-4$$
$$=-1$$

$$C_{23}=-\begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix}$$
$$=-\left[1\times 2-3\times(-1)\right]$$
$$=-(2+3)$$
$$=-5$$

Take the product of C_{31} and C_{23}.

$$\therefore C_{31}\cdot C_{23}=-1\times(-5)$$
$$=5$$

Hence, the correct answer is option (a).

Solution 14

If a function y is decreasing in the interval (*a*, *b*) then $\frac{dy}{dx}<0$ for *x* ∊ (*a*, *b*).

Differentiate $y=x^2e^{-x}$ with respect to *x*.

$$\frac{dy}{dx}=\frac{d\left(x^2e^{-x}\right)}{dx}$$
$$=2xe^{-x}-x^2e^{-x}$$
$$=xe^{-x}(2-x)$$

Since $e^{-x}>0$ for all *x*, the sign of first derivative depends on $x(2-x)$.

For *x* < 0, the value of $x(2-x)$ is negative since $2-x>0$. So, the product $x(2-x)$ is negative.

For *x* > 2, the value of $x(2-x)$ is negative since $2-x<0$. So, the product $x(2-x)$ is negative.

Thus, the function is decreasing in the interval $(-\infty,\ 0)\cup(2,\ \infty)$.

Hence, the correct answer is option (d).

Solution 15

$$x^2+y^2\le 4$$
$$\Rightarrow y^2\le 4-x^2$$

The value of y^2 will always be a positive value so, the value of $4-x^2$ must be greater than or equal to zero.

$\therefore 4-x^2 \geq 0$
$\Rightarrow x^2 \leq 4$
$\Rightarrow -2 \leq x \leq 2$

Since the given relation in set **Z**, the domain of R is {−2, −1, 0, 1, 2}.

Hence, the correct answer is option (b).

Solution 16

The system of linear equations given by $AX=B$ is consistent if $|A| \neq 0$.

$\therefore \begin{vmatrix} 5 & k \\ 3 & 3 \end{vmatrix} \neq 0$

$\Rightarrow 15-3k \neq 0$

$\Rightarrow 3k \neq 15$

$\Rightarrow k \neq 5$

Hence, the correct answer is option (d).

Solution 17

On *x*-axis, *y* = 0.
$y\left(1+x^2\right)=2-x$

$\Rightarrow 0 \times \left(1+x^2\right)=2-x$

$\Rightarrow 2-x=0$

$\Rightarrow x=2$

So, the equation of the curve when *y* = 0 gives *x* = 2.

Differentiate $y=\frac{2-x}{(1+x^2)}$ with respect to *x* and substitute *x* = 2.

$$\left.\frac{dy}{dx}\right|_{x=2}=\left.\frac{d\left(\frac{2-x}{1+x^2}\right)}{dx}\right|_{x=2}$$

$$=\left.\frac{-\left(1+x^2\right)-2x(2-x)}{\left(1+x^2\right)^2}\right|_{x=2}$$

$$=\frac{-\left(1+2^2\right)-2\times2(2-2)}{\left(1+2^2\right)^2}$$

$$=\frac{-5-0}{5^2}$$

$$=-\frac{1}{5}$$

The equation of the tangent of slope $-\frac{1}{5}$ at point (2, 0):

$y-0=-\frac{1}{5}(x-2)$

$\Rightarrow 5y=-x+2$

$\Rightarrow x+5y=2$

Hence, the correct answer is option (c).

Solution 18

$$\begin{bmatrix} 3c+6 & a-d \\ a+d & 2-3b \end{bmatrix}=\begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$$

On comparing both sides:

$3c+6=12 \qquad .....(1)$

$a-d=2 \qquad .....(2)$

$a+d=-8 \qquad .....(3)$

$2-3b=-4 \qquad .....(4)$

From (1),
$3c=12-6$

$\Rightarrow c=2$

From (4),
$-3b=-4-2$

$\Rightarrow -3b=-6$

$\Rightarrow b=2$

Add (2) and (3).

$a-d+a+d=2-8$
$\Rightarrow 2a=-6$
$\Rightarrow a=-3$
$\therefore -3-d=2 \qquad \text{[From (2)]}$
$\Rightarrow d=-5$

Substitute the values of *a*, *b*, *c* and *d* in $ab-cd$.
$$\begin{aligned}\therefore ab-cd &= -3\times 2-2\times(-5)\\ &= -6+10\\ &= 4\end{aligned}$$

Hence, the correct answer is option (a).

Solution 19

$$\begin{aligned}\tan^{-1}\left(\tan\frac{9\pi}{8}\right) &= \tan^{-1}\left(\tan\left(\pi+\frac{\pi}{8}\right)\right)\\ &= \tan^{-1}\left(\tan\left(\frac{\pi}{8}\right)\right) \qquad [\because \tan(\pi+x)=\tan(\\ &= \frac{\pi}{8}\end{aligned}$$

Hence, the correct answer is option (a).

$$\begin{aligned}\tan\left(\frac{9\pi}{8}\right) &= \tan\left(\pi+\frac{\pi}{8}\right)\\ &= \tan\left(\frac{\pi}{8}\right) \qquad .....(1)\end{aligned}$$

$\text{Let } \tan\left(\frac{\pi}{8}\right)=x \qquad .....(2)$
$\Rightarrow \tan^{-1}x=\frac{\pi}{8}$
$\Rightarrow \tan^{-1}\left(\tan\frac{\pi}{8}\right)=\frac{\pi}{8} \qquad \text{[From (2)]}$
$\Rightarrow \tan^{-1}\left(\tan\frac{9\pi}{8}\right)=\frac{\pi}{8} \qquad \text{[From (1)]}$

Solution 20

$$P = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$Q^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$\Rightarrow \left(Q^T\right)^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}^T$$

$$\Rightarrow Q = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

Subtract Q from P.

$$P - Q = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3-(-1) & 4-1 \\ -1-2 & 2-2 \\ 0-1 & 1-3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

Hence, the correct answer is option (b).

Section B

Solution 21

Given: $f(x) = 2x^3 - 15x^2 + 36x + 6$

$\Rightarrow f'(x) = 6x^2 - 30x + 36$

$\Rightarrow f'(x) = 6\left(x^2 - 5x + 6\right)$

$\Rightarrow f'(x) = 6\left(x^2 - 2x - 3x + 6\right)$

$\Rightarrow f'(x) = 6(x-2)(x-3)$

$\therefore f'(x) = 0$

$\Rightarrow (x-2)(x-3) = 0$

$\Rightarrow x = 2 \text{ and } x = 3$

The points x = 2 and x = 3 divide the real line into three disjoint intervals i.e., $(-\infty,\ 2],\ \ (2,\ 3),\ [3,\ \infty)$.

In the intervals $(-\infty,\ 2]$ and $[3,\ \infty)$, $f'(x) \geq 0$.

Thus, $f(x)$ is increasing in $(-\infty, 2] \cup [3, \infty)$.

Hence, the correct answer is option (c).

Solution 22

$$x = 2\cos\theta - \cos 2\theta$$
$$\Rightarrow \frac{dx}{d\theta} = \frac{d}{d\theta}(2\cos\theta - \cos 2\theta)$$
$$\Rightarrow \frac{dx}{d\theta} = -2\sin\theta - (-2\sin 2\theta)$$
$$\Rightarrow \frac{dx}{d\theta} = -2\sin\theta + 2\sin 2\theta$$

And, $y = 2\sin\theta - \sin 2\theta$
$$\Rightarrow \frac{dy}{d\theta} = \frac{d}{d\theta}(2\sin\theta - \sin 2\theta)$$
$$\Rightarrow \frac{dy}{d\theta} = 2\cos\theta - (2\cos 2\theta)$$
$$\Rightarrow \frac{dy}{d\theta} = 2\cos\theta - 2\cos 2\theta$$

$$\therefore \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$
$$= \frac{2\cos\theta - 2\cos 2\theta}{-2\sin\theta + 2\sin 2\theta}$$
$$= \frac{\cos\theta - \cos 2\theta}{-\sin\theta + \sin 2\theta}$$
$$= \frac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta}$$

Hence, the correct answer is option (b).

Solution 23

The domain of $\cos^{-1}x$ is $[-1, 1]$.

Thus, domain of $\cos^{-1}(2x - 3)$ is

$$2x-3\in[-1,\ 1]$$
$$\Rightarrow -1\le 2x-3\le 1$$
$$\Rightarrow -1+3\le 2x\le 1+3$$
$$\Rightarrow 2\le 2x\le 4$$
$$\Rightarrow 1\le x\le 2$$
$$\Rightarrow x\in[1,\ 2]$$

Hence, the correct answer is option (d).

Solution 24

Given: $a_{ij}=\begin{cases}2i+3j, & i<j\\ 5, & i=j\\ 3i-2j, & i>j\end{cases}$

$$\therefore \text{A}=\begin{bmatrix}5 & 2\times1+3\times2 & 2\times1+3\times3\\ 3\times2-2\times1 & 5 & 2\times2+3\times3\\ 3\times3-2\times1 & 3\times3-2\times2 & 5\end{bmatrix}$$
$$=\begin{bmatrix}5 & 2+6 & 2+9\\ 6-2 & 5 & 4+9\\ 9-2 & 9-4 & 5\end{bmatrix}$$
$$=\begin{bmatrix}5 & 8 & 11\\ 4 & 5 & 13\\ 7 & 5 & 5\end{bmatrix}$$

Therefore, the number of elements in A which are more than 5 are 4.

Hence, the correct answer is option (b).

Solution 25

The given function *f* is $f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x}, & \text{if } x\ne\dfrac{\pi}{2}\\ 3, & \text{if } x=\dfrac{\pi}{2}\end{cases}$

The given function *f* is continuous at $x=\frac{\pi}{2}$, if *f* is defined at $x=\frac{\pi}{2}$ and if the value of the *f* at $x=\frac{\pi}{2}$ equals the limit of *f* at $x=\frac{\pi}{2}$.

It is evident that *f* is defined at $x=\frac{\pi}{2}$ and $f\left(\frac{\pi}{2}\right)=3$

$$\lim_{x \to \frac{\pi}{2}} f(x) = \lim_{x \to \frac{\pi}{2}} \frac{k \cos x}{\pi - 2x}$$

Put $x = \frac{\pi}{2} + h$

Then, $x \to \frac{\pi}{2} \Rightarrow h \to 0$

$$\therefore \lim_{x \to \frac{\pi}{2}} f(x) = \lim_{x \to \frac{\pi}{2}} \frac{k \cos x}{\pi - 2x} = \lim_{h \to 0} \frac{k \cos\left(\frac{\pi}{2} + h\right)}{\pi - 2\left(\frac{\pi}{2} + h\right)}$$

$$= k \lim_{h \to 0} \frac{-\sin h}{-2h} = \frac{k}{2} \lim_{h \to 0} \frac{\sin h}{h} = \frac{k}{2}.1 = \frac{k}{2}$$

$$\therefore \lim_{x \to \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \frac{k}{2} = 3$$

$$\Rightarrow k = 6$$

Therefore, the required value of *k* is 6.

Hence, the correct answer is option (c).

Solution 26

Given: $X = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$$\therefore X^2 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow X^2 - X = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= 2I$$

Hence, the correct answer is option (a).

Solution 27

Let X = $\{x^2 : x \in \mathbf{N}\}$ and $f : \mathbf{N} \rightarrow X$ such that $f(x) = x^2$, $x \in \mathbf{N}$.
Let x, $y \in \mathbf{N}$ such that $f(x) = f(y)$.
$\Rightarrow x^2 = y^2$
$\Rightarrow x = y$
So, f is a one-one function.
Let $y \in \mathbf{X}$.

$$\Rightarrow f(x) = y$$

$$\Rightarrow x^2 = y$$

$$\Rightarrow x = \sqrt{y} \in \mathbf{N} \quad \left[\because y \in \mathrm{X} = \left\{x^2 : x \in \mathbf{N}\right\}\right]$$

So, f is onto function
Therefore, f is a bijective function.

Hence, the correct answer is option (d).

Solution 28

Given: Z = $2x + 5y$
At P, Z = 2 × 0 + 5 × 5 = 25
At Q, Z = 2 × 1 + 5 × 5 = 27
At R, Z = 2 × 4 + 5 × 2 = 18
At S, Z = 2 × 12 + 5 × 0 = 24

Thus, the minimum value of Z is at the point R.

Hence, the correct answer is option (c).

Solution 29

The equation of the given curve is $ay^2 = x^3$.
On differentiating with respect to x, we have:

$2ay\frac{\mathrm{d}\,y}{\mathrm{d}\,x} = 3x^2$

$\Rightarrow \frac{\mathrm{d}\,y}{\mathrm{d}\,x} = \frac{3x^2}{2ay}$

The slope of a tangent to the curve at (x_0, y_0) is $\left.\frac{\mathrm{d}\,y}{\mathrm{d}\,x}\right|_{(x_0,\, y_0)}$.

Thus, the slope of the tangent to the given curve at (am^2, am^3) is

$\left.\frac{\mathrm{d}\,y}{\mathrm{d}\,x}\right|_{(am^2,\, am^3)} = \frac{3\left(am^2\right)^2}{2a(am^3)} = \frac{3m}{2}$

So, slope of normal at (am^2, am^3) = $\frac{-1}{\text{Slope of the tangent at } (am^2, am^3)} = \frac{-2}{3m}$

Hence, the equation of the normal at (am^2, am^3) is given by,

$y - am^3 = \frac{-2}{3m}\left(x - am^2\right)$

$\Rightarrow 3my - 3am^4 = -2x + 2am^2$

$\Rightarrow 2x + 3my - 3am^4 - 2am^2 = 0$

Hence, the correct answer is option (d).

Solution 30

Given: A is a square matrix of order 3 and |A| = –5

$|\text{adj } A| = |A|^{3-1}$ $\left(\because |\text{adj } X| = |X|^{n-1}, \text{ where } n \text{ is the order o}\right.$

$\Rightarrow |\text{adj } A| = |A|^2$

$\Rightarrow |\text{adj } A| = (-5)^2$ $(\text{Given : } |A| = 5\ )$

$\Rightarrow |\text{adj } A| = 25$

Hence, the correct answer is option (c).

Solution 31

$\text{Put } x=\cos 2\theta \text{ so that } \theta=\frac{1}{2}\cos^{-1} x. \text{ Then, we have :}$

$\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

$=\tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$

$=\tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}-\sqrt{2Sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right)$

$=\tan^{-1}\left(\frac{\sqrt{2}\cos\theta-\sqrt{2}Sin\theta}{\sqrt{2}\cos\theta+\sqrt{2}Sin\theta}\right)$

$=\tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$

$=\tan^{-1}(1)-\tan^{-1}(\tan\theta)$ $\quad \left[\tan^{-1}\left(\frac{x-y}{1+xy}\right)=\tan^{-1}x-\tan^{-1}y\right]$

$=\frac{\pi}{4}-\theta$

$=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$

Hence, the correct answer is option (c).

Solution 32

Given: $A=\begin{bmatrix}\alpha & -2\\ -2 & \alpha\end{bmatrix}$ and $|A^3|=125$

$A=\begin{bmatrix}\alpha & -2\\ -2 & \alpha\end{bmatrix}$

$\Rightarrow \ |A|=\begin{vmatrix}\alpha & -2\\ -2 & \alpha\end{vmatrix}=\alpha^2-4$

Also, $|A^3|=125$

$\Rightarrow |A|^3=125 \qquad (\because |X^n|=|X|^n)$

$\Rightarrow |A|^3=(5)^3$

$\Rightarrow |A|=5$

$\Rightarrow \alpha^2-4=5$

$\Rightarrow \alpha^2=9$

$\Rightarrow \alpha=\pm 3$

Hence, the correct answer is option (a).

Solution 33

$y=\sin\left(m\ \sin^{-1}x\right)$

Differentiating both sides with respect to *x*, we get

$\frac{\mathrm{d}y}{\mathrm{d}x}=\cos\left(m\ \sin^{-1}x\right)\times\frac{m}{\sqrt{1-x^2}}$

$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{m\left[\cos\left(m\ \sin^{-1}x\right)\right]}{\sqrt{1-x^2}}$

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\left\{\frac{\left(\sqrt{1-x^2}\right)\left(-\sin\left(m\left(\sin^{-1}x\right)\right)\right)\times\frac{m}{\sqrt{1-x^2}}-\left(m\cos\left(m\left(\ \sin^{-1}x\right)\right)\right)\times\frac{1}{2\sqrt{1-x^2}}\times(-2x)}{\left(\sqrt{1-x^2}\right)^2}\right\}$$

$$\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=m\left\{\frac{-m\ \sin\left(m\left(\sin^{-1}x\right)\right)+\cos\left(m\left(\ \sin^{-1}x\right)\right)\times\frac{x}{\sqrt{1-x^2}}}{1-x^2}\right\}$$

$$\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=m\left\{\frac{-m\ \sin\left(m\left(\sin^{-1}x\right)\right)}{1-x^2}+\frac{x\ \cos\left(m\left(\ \sin^{-1}x\right)\right)}{(1-x^2)\sqrt{1-x^2}}\right\}$$

$$\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=m\left\{\frac{-m\ \sin\left(m\left(\sin^{-1}x\right)\right)}{1-x^2}+\frac{\frac{x}{m}\frac{\mathrm{d}y}{\mathrm{d}x}}{(1-x^2)}\right\}\qquad\left[\because\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{m\cos\left(m\left(\ \sin^{-1}x\right)\right.}{\sqrt{1-x^2}}\right.$$

$$\Rightarrow\left(1-x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=m\left\{-m\ \sin\left(m\left(\sin^{-1}x\right)\right)+\frac{x}{m}\frac{\mathrm{d}y}{\mathrm{d}x}\right\}$$

$$\Rightarrow\left(1-x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-m^2\ \sin\left(m\left(\sin^{-1}x\right)\right)+x\frac{\mathrm{d}y}{\mathrm{d}x}\qquad\left[\because y=\sin\left(m\left(\right.\right.\right.$$

$$\Rightarrow\left(1-x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-m^2\ y+x\frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\Rightarrow\left(1-x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-x\frac{\mathrm{d}y}{\mathrm{d}x}+m^2\ y=0$$

Hence, the correct answer is option (b).

Solution 34

Let $\tan^{-1}\left(\sqrt{3}\right)=x$.

Then, $\tan x=\sqrt{3}=\tan\frac{\pi}{3},\ \text{where}\ \frac{\pi}{3}\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

$\therefore\tan^{-1}\left(\sqrt{3}\right)=\frac{\pi}{3}$

Let $\cot^{-1}\left(-\sqrt{3}\right)=y$.

Then

$\cot y=-\sqrt{3}=-\cot\left(\frac{\pi}{6}\right)=\cot\left(\pi-\frac{\pi}{6}\right)=\cot\left(\frac{5\pi}{6}\right),\ \text{where}\ \frac{\pi}{3}\in(0,\pi)$.

The range of the principal value branch of $\cot^{-1}$ is $(0, \pi)$.

$\therefore \cot^{-1}\left(-\sqrt{3}\right) = \frac{5\pi}{6}$

$\therefore \tan^{-1}\left(\sqrt{3}\right) - \cot^{-1}\left(-\sqrt{3}\right) = \frac{\pi}{3} - \frac{5\pi}{6} = \frac{2\pi - 5\pi}{6} = \frac{-3\pi}{6} = \frac{-\pi}{2}$

Hence, the correct answer is option (b).

Solution 35

Let $y = \left(\frac{1}{x}\right)^{x} \qquad \ldots\ldots(1)$

Taking Log on both sides, we get

$\log y = -x \log x$

Differentiating both sides with respect to *x*, we get

$\frac{1}{y}\frac{dy}{dx} = -1 - \log x$

$\Rightarrow \frac{dy}{dx} = y(-1 - \log x) \qquad \ldots\ldots(2)$

Equating $\frac{dy}{dx} = 0$, we get

$y(-1 - \log x) = 0$

$\Rightarrow \log x = -1$

$\Rightarrow x = e^{-1}$

Differentiating (2) on both sides with respect to *x*, we get

$\Rightarrow \frac{dy}{dx} = -y - y \log x$

$\Rightarrow \frac{d^2y}{dx^2} = -\frac{dy}{dx} - \left(\log x \frac{dy}{dx} + \frac{1}{x}\right)$

$\Rightarrow \frac{d^2y}{dx^2} = -\left(\frac{dy}{dx} + \log x \frac{dy}{dx} + \frac{1}{x}\right)$

$\Rightarrow \frac{d^2y}{dx^2} = -\text{ve} < 0$

For maximum $\frac{d^2y}{dx^2} < 0$, thus maximum is attained at $x = e^{-1}$

For maximum value of x put $x = e^{-1}$ in (2),

$y = \left(\frac{1}{e^{-1}}\right)^{e^{-1}}$

$\Rightarrow y = e^{\left(\frac{1}{e}\right)}$

Hence, the correct answer is option (a).

Solution 36

$\mathrm{Y} = [m_{ij}]$

$$\mathrm{m}_{11} = \begin{vmatrix} 4 & -5 \\ -1 & 3 \end{vmatrix} = 7,\ \mathrm{m}_{12} = \begin{vmatrix} 3 & -5 \\ 2 & 3 \end{vmatrix} = 19,\ \ \mathrm{m}_{13} = \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = -11,$$

$$\mathrm{m}_{21} = \begin{vmatrix} -1 & 2 \\ -1 & 3 \end{vmatrix} = -1,\ \mathrm{m}_{22} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1,\ \mathrm{m}_{23} = \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = 1,$$

$$\mathrm{m}_{31} = \begin{vmatrix} -1 & 2 \\ 4 & -5 \end{vmatrix} = -3,\ \mathrm{m}_{22} = \begin{vmatrix} 1 & 2 \\ 3 & -5 \end{vmatrix} = -11,\ \mathrm{m}_{33} = \begin{vmatrix} 1 & -1 \\ 3 & 4 \end{vmatrix} = 7$$

Thus, the matrix $\mathrm{Y} = [\mathrm{m_{ij}}] = \begin{bmatrix} 7 & 19 & -11 \\ -1 & -1 & 1 \\ -3 & -11 & 7 \end{bmatrix}$

Hence, the correct answer is option (d).

Solution 37

Given: *f*: **R** → **R** defined by $f(x) = 2 + x^2$

For One-One:

Let x and y be two arbitrary elements of **R** such that $f(x) = f(y)$.

Then, $f(x) = f(y)$

$\Rightarrow 2 + x^2 = 2 + y^2$

$\Rightarrow x^2 = y^2$

$\Rightarrow x = \pm y$

Here, $f(x) = f(y)$ does not provide the unique solution $x = y$ but it provides $x = \pm y$.

Thus, *f* is not a one-one function.

For Onto:

Clearly $f(x) = 2 + x^2 \geq 2$ for all $x \in$ **R**.

So, negative real numbers in **R**(co-domain) do not have their pre-images in **R**(domain).

Thus, *f* is not an onto function.

Therefore, *f* is neither one-one nor onto.

Hence, the correct answer is option (d).

Solution 38

The corner points of the feasible region are A(0, 10), B(12, 6), C(20, 0) and O(0, 0).

Corner points	$Z = 2x - y + 5$

A(0, 10)	*Z* = −5
B(12, 6)	*Z* = 23
C(20, 0)	*Z* = 45
O(0, 0)	*Z* = 5

Thus, the minimum value of *Z* is −5.

Hence, the correct answer is option (b).

Solution 39

Given: x = −4 is a root of $\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0,$

$$\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$$
$$\Rightarrow x\left(x^2 - 2\right) - 2\left(x - 3\right) + 3\left(2 - 3x\right) = 0$$
$$\Rightarrow x^3 - 2x - 2x + 6 + 6 - 9x = 0$$
$$\Rightarrow x^3 - 4x + 12 - 9x = 0$$
$$\Rightarrow x^3 - 13x + 12 = 0 \qquad \ldots\ldots(1)$$

Since, x = −4 is a root, thus (x + 4) will be a factor of the equation $x^3 - 13x + 12 = 0.$

$x^2 - 4x + 3$

$x + 4 \,)\, x^2 \quad - 13x + 12 \,($

$ x^3 + 4x^2$

$ - \quad -$

$- 4x^2 - 13x$

$- 4x^2 - 16x + 12$

$+ \quad +$

$3x + 12$

$3x + 12$

$- \quad -$

0

$\text{So, } x^3 - 13x + 12 = (x+4)\left(x^2 - 4x + 3\right) \qquad \ldots\ldots(2)$

$\text{From } (1) \text{ and } (2), \text{ we get}$

$\Rightarrow (x+4)\left(x^2 - x - 3x + 3\right) = 0$

$\Rightarrow (x+4)\left[x(x-1) - 3(x-1)\right] = 0$

$\Rightarrow (x+4)(x-1)(x-3) = 0$

$\Rightarrow (x+4) = 0 \text{ or } (x-1) = 0 \text{ or } (x-3) = 0$

$\Rightarrow x = -4 \text{ or } x = 1 \text{ or } x = 3$

Thus, the other two roots are 1 and 3.

Therefore, the sum of the other two roots = 1 + 3 = 4.

Hence, the correct answer is option (a).

Solution 40

Differentiate f(x) with respect to x.

$f'(x) = 4 - \frac{1}{2} \times 2x$

$= 4 - x$

Thus, $f'(x)$ is 0 at x = 4.

Since $f'(x)$ is not undefined at any value of x, there is only one critical point.

Evaluating the value of f(x) at critical point x = 4 and end points x = −2 and $x = \frac{9}{2}$.

$f(4) = 4(4) - \frac{1}{2}(4)^2$

$= 16 - 8$

$= 8$

$f(-2) = 4(-2) - \frac{1}{2}(-2)^2$

$= -8 - 2$

$= -10$

$$f\left(\frac{9}{2}\right)=4\left(\frac{9}{2}\right)-\frac{1}{2}\left(\frac{9}{2}\right)^2$$
$$=18-\frac{81}{8}$$
$$=\frac{144-81}{8}$$
$$=\frac{63}{8}$$
$$=7.875$$

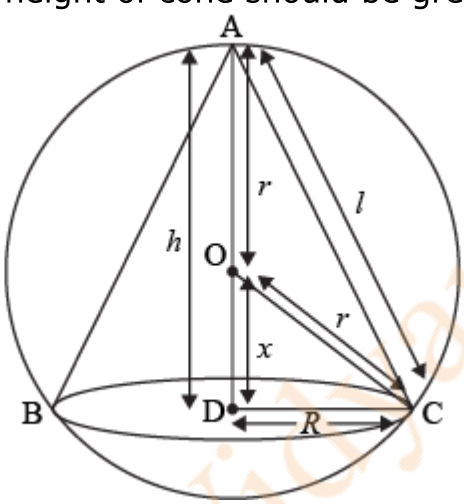
Thus, the absolute maximum value of the function $f(x)=4x-\frac{1}{2}x^2$ in the interval $\left[-2,\ \frac{9}{2}\right]$ is 8.

Hence, the correct answer is option (a).

Section C

Solution 41

Given that a sphere of radius *r* and a right circular cone of height *h*.
Let *R* be the radius of the cone and OD = *x*.
For maximum curved surface area, the edges of cone must touch sphere and height of cone should be greater than radius of sphere.



Square of curved surface area = $(\pi Rl)^2$ .....(1)
In ΔADC,
$l=\sqrt{R^2+(r+x)^2}$ (By Pythagoras in ΔADC)

$$=\sqrt{r^2-x^2+r^2+x^2+2rx}$$
$$=\sqrt{2r^2+2rx}$$
$$=\sqrt{2r}\sqrt{r+x} \qquad \ldots..(2)$$

From (1) and (2), we get

$$(\pi Rl)^2 = \pi^2 R^2 l^2$$
$$= \pi^2 \left(r^2 - x^2\right) \left(\sqrt{2r}\sqrt{r+x}\right)^2$$
$$= \pi^2 \left(r^2 - x^2\right) \left(2r\left(r+x\right)\right)$$
$$= 2\pi^2 r\left(r^2 - x^2\right)\left(r + x\right)$$
$$= 2\pi^2 rh\left(r^2 - x^2\right) \qquad (\because r + x = h)$$
$$= 2\pi^2 rh\left(r^2 - (h-r)^2\right)$$
$$= 2\pi^2 rh\left(r^2 - \left(h^2 + r^2 - 2hr\right)\right)$$
$$= 2\pi^2 rh\left(r^2 - h^2 - r^2 + 2hr\right)$$
$$= 2\pi^2 r\left(2h^2 r - h^3\right)$$

Hence, the correct answer is option (c).

Solution 42

Given: Z = $ax + 2by$

The maximum value of Z occurs at Q(3, 5) and S(4, 1)

∴ Z at Q = Z at S

⇒ $3a + 10b = 4a + 2b$

⇒ $a - 8b = 0$

Hence, the correct answer is option (d).

Solution 43

Given:

$$y^2 = 4x \quad \ldots\ldots(1)$$
$$xy = c \quad \ldots\ldots(2)$$

From (1) and (2), we get

$y^2 = 4 \times \frac{c}{y}$

$\Rightarrow y^3 = 4c$

$\Rightarrow y = (4c)^{\frac{1}{3}}$

and,

$x \times (4c)^{\frac{1}{3}} = c$

$\Rightarrow x = \frac{c}{(4c)^{\frac{1}{3}}}$

$\Rightarrow x = \left(\frac{c}{2}\right)^{\frac{2}{3}}$

On differentiating (1) with respect to *x*, we get

$2y\frac{\mathrm{d}y}{\mathrm{d}x} = 4$

$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{y}$

$\Rightarrow m_1 = \frac{2}{(4c)^{\frac{1}{3}}}$

On differentiating (2) with respect to *x*, we get

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-c}{x^2}$

$\Rightarrow m_2 = \frac{-c}{\left(\frac{c}{2}\right)^{\frac{4}{3}}}$

$\Rightarrow m_2 = -\frac{2^{\frac{4}{3}}}{c^{\frac{1}{3}}}$

Since curves (1) and (2) intersect at right angles, so $m_1 \times m_2 = -1$.

$\Rightarrow \frac{2}{(4c)^{\frac{1}{3}}} \times \frac{-2^{\frac{4}{3}}}{c^{\frac{1}{3}}} = -1$

$\Rightarrow 2^{1+\frac{4}{3}-\frac{2}{3}} = c^{\frac{2}{3}}$

$\Rightarrow 2^{\frac{5}{3}} = c^{\frac{2}{3}}$

$\Rightarrow 2^5 = c^2$

$\Rightarrow c = \pm\sqrt{32} = \pm 4\sqrt{2}$

Hence, the correct answers are (a) and (d).

Disclaimer: Both (a) and (d) are correct.

Solution 44

$$X = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

|X| = 2(3 × 4) = 24 ≠ 0

So, X is invertible.

$$adj\,(X) = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$X^{-1} = \frac{1}{|X|} adj\,(X)$$

$$\Rightarrow X^{-1} = \frac{1}{24} \begin{bmatrix} 12 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$\Rightarrow X^{-1} = \begin{bmatrix} \frac{12}{24} & 0 & 0 \\ 0 & \frac{8}{24} & 0 \\ 0 & 0 & \frac{6}{24} \end{bmatrix}$$

$$\Rightarrow X^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$$

Hence, the correct answer is option (d).

Solution 45

The corner points of the feasible region are P(0, 40), Q(30, 20) and R(40, 0).

Corner points	**Z = 4*x* + 3*y***
P(0, 40)	Z = 120
Q(30, 20)	Z = 180
R(40, 0)	Z = 160

The maximum value of Z is at Q(30, 20).

Hence, the correct answer is option (b).

Solution 46

The total cost of the pit is the cost charged by local authorities for the space and the cost charged by the labourer.

Given, the side of square plot is x m and depth of the pit is h m.

The cost charge by local authorities for the space is ₹50 per square meter.

So, the total cost charged by the local authorities for the square plot
= ₹$(50 \times x^2)$ .....(1)

Total digging is of 250 m^3.

∴ Volume of the pit = 250 m^3

$\Rightarrow x^2 \times h = 250$

$\Rightarrow x^2 = \frac{250}{h}$

Form (1), we get

Total cost charged by the local authorities for the square plot = $\frac{12500}{h}$

It is given that labourer charged ₹400 × $(\text{depth})^2$, i.e., ₹$400h^2$

∴ Total cost, $c = \frac{12500}{h} + 400h^2$

Hence, the correct answer is option (b).

Solution 47

$$c = \frac{12500}{h} + 400h^2$$

$$\Rightarrow \frac{dc}{dh} = -\frac{12500}{h^2} + 800h$$

$$\therefore \frac{dc}{dh} = 0$$

$$\Rightarrow -\frac{12500}{h^2} + 800h = 0$$

$$\Rightarrow \frac{12500}{h^2} = 800h$$

$$\Rightarrow h^3 = \frac{12500}{800} = \frac{125}{8} = \left(\frac{5}{2}\right)^3$$

$$\Rightarrow h = \frac{5}{2} = 2.5 \text{ m}$$

Hence, the correct answer is option (c).

Solution 48

$c = \frac{12500}{h} + 400h^2$

$\Rightarrow \frac{dc}{dh} = -\frac{12500}{h^2} + 800h$

$\Rightarrow \frac{d^2c}{dh^2} = \frac{25000}{h^3} + 800$

Hence, the correct answer is option (a).

Solution 49

Total cost, $c = \frac{12500}{h} + 400h^2$ .....(1)

$\therefore$ Volume of the pit = 250 m^3

$\Rightarrow x^2 \times h = 250$

$\Rightarrow h = \frac{250}{x^2}$ .....(2)

From (1) and (2), we get

$c = 50x^2 + 400\left(\frac{250}{x^2}\right)^2$

$\Rightarrow c = 50x^2 + 400\left(\frac{62500}{x^4}\right)$

$\Rightarrow \frac{\mathrm{d}c}{\mathrm{d}x} = 100x + 400 \times 62500 \times \frac{(-4)}{x^5}$

To find the minimum cost, we put $\frac{dc}{dx} = 0$

$100x + 400 \times 62500 \times \frac{(-4)}{x^5} = 0$

$\Rightarrow 100x = 400 \times 62500 \times \frac{4}{x^5}$

$\Rightarrow x^6 = 4 \times 4 \times 62500$

$\Rightarrow x = 10 \text{ m}$

Now, to check at x = 10 m, the cost is minimum or maximum,

$\frac{d^2c}{dx^2} = 100 + 400 \times 62500 \times 4 \times \frac{5}{x^6} > 0$

Here, $\frac{d^2c}{dx^2} > 0$

So, at *x* = 10 m the cost is minimum.

Hence, the correct answer is option (d).

Solution 50

The total cost, $c = 50x^2 + 400\left(\frac{62500}{x^4}\right)$

The cost is minimum at *x* = 10 m

So, at *x* = 10 m, *c* = 7500

$c = 50(10)^2 + 400\left(\frac{62500}{10^4}\right)$

$\Rightarrow c = 5000 + 4 \times 625$

$\Rightarrow c = 7500$

Hence, the correct answer is option (b).